The Stationary Boltzmann equation for a two component gas in the slab with different molecular masses.

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Abstract

The stationary Boltzmann equation for hard and soft forces in the context of a two component gas is considered in the slab when the molecular masses of the 2 component are different. An L^1 existence theorem is proved when one component satisfies a given indata profile and the other component satisfies diffuse reflection at the boundaries. Weak L^1 compactness is extracted from the control of the entropy production term of the mixture.

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1 Introduction and setting of the problem.

This article is devoted to the proof of an existence theorem for the stationary Boltzmann equation in the situation of a two component gas having different molecular masses for the geometry of the slab. The slab being represented by the interval $[-1, 1]$, the Boltzmann equation reads

$$\xi \frac{\partial}{\partial x} f_A(x, v) = Q_{AA}(f_A, f_A)(x, v) + Q_{AB}(f_A, f_B)(x, v), \tag{1.1}$$

$$\xi \frac{\partial}{\partial x} f_B(x, v) = Q_{BB}(f_B, f_B)(x, v) + Q_{BA}(f_B, f_A)(x, v), \tag{1.2}$$

$$x \in [-1, 1],\ v \in \mathbb{R}^3.$$

The non-negative functions f_A and f_B represent the distribution functions of the A and the B component with x the position and v the velocity. ξ is the velocity component in the x direction. For for any $\alpha,\, \beta\, \in \{A, B\}$, $Q_{\alpha,\beta}$ corresponds to the non linear Boltzmann collision operator between the species α and β. More precisely, it is defined for any $\{\alpha, \beta\} \in \{A, B\}$ by

$$Q_{\alpha,\beta}(v) = \int_{\mathbb{R}^3\times\mathcal{S}^2} \mathcal{B}^{\alpha,\beta} \left(f_\alpha(x, v'_*) f_\beta(x, v') - f_\beta(x, v_*) f_\alpha(x, v) \right) d\omega dv_* \quad (1.3)$$

where

$$v'^{(\beta\alpha)} = v + \frac{2m^\beta}{m^\alpha + m^\beta} \langle v_* - v, \omega\rangle\omega, \qquad v'^{(\beta\alpha)}_* = v_* - \frac{2m^\beta}{m^\alpha + m^\beta} \langle v_* - v, \omega\rangle\omega. \quad (1.4)$$

In the formula (1.4), $v'^{(\beta\alpha)}$ and $v'^{(\beta\alpha)}_*$ represent the post-colisional velocities between the species α and β and m^α is the mass of the specy α. For more precisions on the model we refer to ([15], [2]).

$\langle \cdot\, , \cdot \rangle$ denotes the Euclidean inner product in $\mathbb{R}^3$. Let ω be represented by the polar angle (with polar axis along $v - v_*$) and the azimutal angle ϕ.

For the sake of clarity, recall the invariant properties of the collision operator $Q_{\alpha,\beta}$, for any $\{\alpha, \beta\} \in \{A, B\}$. For more details we refer to ([17]).

Property 1.1. *For $\alpha, \beta \in \{A, B\}$, with $\alpha \neq \beta$, it holds that*

$$\int_{\mathbb{R}^3} (1, m^\alpha v, m^\alpha |v|^2) Q_{\alpha,\alpha}(f_\alpha, f_\alpha) dv = 0,$$

$$\int_{\mathbb{R}^3} Q_{\alpha,\beta}(f_\alpha, f_\beta) dv = 0,$$

$$\int_{\mathbb{R}^3} m^\alpha v\, Q_{\alpha,\beta}(f_\alpha, f_\beta) dv + \int_{\mathbb{R}^3} m^\alpha v\, Q_{\beta,\alpha}(f_\beta, f_\alpha) dv = 0,$$

$$\int_{\mathbb{R}^3} m^\alpha v^2\, Q_{\alpha,\beta}(f_\alpha, f_\beta) dv + \int_{\mathbb{R}^3} m^\alpha v^2\, Q_{\beta,\alpha}(f_\beta, f_\alpha) dv = 0.$$

The function $\mathcal{B}^{\alpha,\beta}(v - v_*, \omega)$ is the collision kernel of $Q_{\alpha,\beta}$. It is a nonnegative function whose form is determined by the molecular interaction. Because of the action and reaction principle, it has the symmetry property $\mathcal{B}^{A,B} = \mathcal{B}^{B,A}$. More precisely, we consider in this paper the following type of kernels

$$\frac{1}{4\sqrt{2\pi}} \left(\frac{d^\alpha + d^\beta}{2} \right)^2 |v - v_*|^\beta b(\theta),$$

with

$$0 \leq \beta < 2, \quad b \in L^1_+([0,2\pi]), \quad b(\theta) \geq c > 0 \quad a.e.$$

for hard forces and

$$-3 \leq \beta < 0, \quad b \in L^1_+([0,2\pi]), \quad b(\theta) \geq c > 0 \quad a.e.$$

for soft forces.

As ([2]) define the collision frequency as the vector (ν_A, ν_B), with for any $\alpha \in \{A, B\}$,

$$\nu_\alpha = \sum_{\beta \in \{A,B\}} \int B^{\alpha,\beta} f_\beta d\omega dv_*.$$

On the boundary of the domain, the two components satisfy different physical properties. Indeed, the A component is supposed to be a condensable gas whereas the B component is supposed to be non condensable.

Hence the boundary conditions for the A component are the given indata profile

$$f_A(-1,v) = kM_-(v), \xi > 0, \qquad f_A(1,v) = kM_+(v), \xi < 0, \tag{1.5}$$

for some positive k. The boundary conditions for the B component are of diffuse reflection type

$$f_B(-1,v) = (\int_{\xi'<0} |\xi'| f_B(-1,v')dv')M_-(v), \quad \xi > 0, \tag{1.6}$$
$$f_B(1,v) = (\int_{\xi'>0} \xi' f_B(1,v')dv')M_+(v), \quad \xi < 0.$$

M_+ and M_- are given normalized Maxwellians

$$M_-(v) = \frac{1}{2\pi T_-^2} e^{-\frac{|v|^2}{2T_-}} \quad \text{and} \quad M_+(v) = \frac{1}{2\pi T_+^2} e^{-\frac{|v|^2}{2T_+}}.$$

As a theoritical point of view, existence theorem for single component gases has been firstly considered. These papers are of interest because the case of the stationary Boltzmann equation is not covered by the DiPerna Lions theory established for the time dependant non linear Boltzmann equation ([16], [14]). In ([6]), an L^1 existence theorem is shown for hard and soft forces when the distribution function has a given indatta profile. In the case of boundary conditions of Maxwell diffuse reflection type, an analogous theorem is also shown in ([7]). In these two papers the solutions are constructed in such a way that they have a given weighted mass. Existence results for the stationary Povzner equation for a bounded domain of $\mathbb{R}^3$ are shown in ([21], [8]). The situation of a two component gas has been considered in ([11], [12]) when the molecular masses of the two gases are the same. The existence theorems are proved for (1.5, 1.6). In these papers, the strategy of the resolution is to use that the sum of the distribution of the two components satisfies the Boltzmann equation for a one component gas. Hence the weak L^1 compactness is firstly obtained for the sum and transmitted to the two distribution functions. But in the present, case due to the different molecular masses, the sum of the distribution functions is not solution to the Boltzmann equation for a single component gas. Therefore the weak L^1 compactenss has to be extracted directy on each component. In ([13]) the situation of a binary mixture close to a local equilibrium is investigated. In that case the solution of the system is constructed as a Hilbert expansion and a rest term rigorously controled. In [17] a moment method is applied in the situation of small Knudsen number to derive a fluid system.

As a physical point of view and as a numerical point of view, a problem of evaporation condensation for a binary mixture far from equilibrium has been considered in ([22]). The binary mixture composed of vapor and non condensable gas in contact with an infinite plane of condensed vapor. Moreover the non condensable gas is supposed to be closed to the condensed vapor. For the numerical simulations the authors used a time-dependant BGK model for a two component gas until a stationary state is reached. The situation of a small Knudsen number has also been investigated in ([1], [4], [3], [25]) where two types of behaviour are pointed out. In a first situation the macroscopic velocity of the two gases tends to zero when the Knudsen number tends to zero. But the zero order term of the temperature is obtained from the first order term of the macroscopic velocity. This means that the macroscopic velocity disappears at the limit but keeps an influence on the limit. This is the ghost effect pointed in ([23]) for a one component gas and in ([1],[4], [3]) for a two component gas. In a second case the B component becomes negligeable and the macroscopic velocity of the A component becomes constent. Moreover the B component accumulates in a thin layer called Knudsen layer at a boundary.

In this paper, weak solutions (f_A, f_B) to the stationary problem in the sense of Definition 1.1 will be considered.

Definition 1.1. *Let M_A and M_B be given nonnegative real numbers.* (f_A, f_B)

is a weak solution to the stationary Boltzmann problem with the β-norms M_A and M_B, if f_A, f_B, ν_A and ν_B $\in L^1_{loc}((-1,1)\times\mathbb{R}^3)$,
$\int(1+|v|)^\beta f_A(x,v)dxdv = M_A$, $\int(1+|v|)^\beta f_B(x,v)dxdv = M_B$, *and there is a constant $k>0$ such that for every test function $\varphi\in C^1_c([-1,1]\times\mathbb{R}^3)$ such that φ vanishes in a neiborhood of $\xi=0$, and on $\{(-1,v);\xi<0\}\cup\{(1,v);\xi>0\}$,*

$$\int_{-1}^1\int_{\mathbb{R}^3}(\xi f_A\frac{\partial\varphi}{\partial x}+Q_{AA}(f_A,f_A)+Q_{AB}(f_A,f_B)\varphi)(x,v)dxdv$$
$$=k\int_{\mathbb{R}^3,\xi<0}\xi M_+(v)\varphi(1,v)dv-k\int_{\mathbb{R}^3,\xi>0}\xi M_-(v)\varphi(-1,v)dv,$$
$$\int_{-1}^1\int_{\mathbb{R}^3}(\xi f_B\frac{\partial\varphi}{\partial x}+Q_{BB}(f_B,f_B)+Q_{BA}(f_B,f_A)\varphi)(x,v)dxdv,$$
$$=\int_{\xi'<0}|\xi|M_+(v)\varphi(1,v)dv(\int_{\xi'>0}\xi' f_B(1,v')dv')$$
$$-\int_{\xi'>0}\xi M_-(v)\varphi(-1,v)dv(\int_{\xi'<0}\xi' f_B(-1,v')dv').$$

Renormalized solutions will also been considered. We recall their definition. Let g be defined for $x>0$ by

$$g(x)=\ln(1+x).$$

Definition 1.2. *Let M_A and M_B be given nonnegative real numbers. (f_A,f_B) is a renormalized solution to the stationary Boltzmann problem with the β-norms M_A and M_B, if f_A, f_B, ν_A, ν_B $\in L^1_{loc}((-1,1)\times\mathbb{R}^3)$,*
$\int(1+|v|)^\beta f_A(x,v)dxdv = M_A$, $\int(1+|v|)^\beta f_B(x,v)dxdv = M_B$, *and there is a constant $k>0$ such that for every test function $\varphi\in C^1_c([-1,1]\times\mathbb{R}^3)$ such that φ vanishes in a neiborhood of $\xi=0$ and on*
$\{(-1,v);\xi<0\}\cup\{(1,v);\xi>0\}$,

$$\int_{-1}^1\int_{\mathbb{R}^3}(\xi g(f_A)\frac{\partial\varphi}{\partial x}+\frac{Q_{AA}(f_A,f_A)}{1+f_A}\varphi+\frac{Q_{AB}(f_A,f_B)}{1+f_A}\varphi)(x,v)dxdv$$
$$=\int_{\mathbb{R}^3,\xi<0}\xi g(kM_+(v))\varphi(1,v)dv-\int_{\mathbb{R}^3,\xi>0}g(\xi kM_-(v))\varphi(-1,v)dv,$$
$$\int_{-1}^1\int_{\mathbb{R}^3}(\xi g(f_B)\frac{\partial\varphi}{\partial x}+\frac{Q_{BB}(f_B,f_A+f_B)}{1+f_B}\varphi+\frac{Q_{BA}(f_B,f_A)}{1+f_B}\varphi)(x,v)dxdv,$$
$$=\int_{\xi<0}\xi g\big((\int_{\xi'>0}\xi' f_B(1,v')dv')M_+(v)\big)\varphi(1,v)dv$$
$$-\int_{\xi>0}\xi g\big(\int_{\xi'<0}\xi' f_B(-1,v')dv')M_-(v)\big)\varphi(-1,v)dv.$$

The main results of this paper are the following theorems

Theorem 1.1. *Given β with $0 \leq \beta < 2$, $M_A > 0$ and $M_B > 0$ there is a weak solution to the stationary problem with β-norms equal to M_A and M_B.*

Theorem 1.2. *Given β with $-3 < \beta < 0$ $M_A > 0$ and $M_B > 0$, there is a renormalized solution to the stationary problem with β-norms equal to M_A and M_B.*

The present paper is organized as follows. The second and the third section are devoted to the proof of the theorems 1.1 and 1.2. In section 2, we perform a fix point step on an approched problem as in ([6], [7], [11], [12]). In the last part we perform a passage to the limit in the sequences of approximation.

2 Approximations with fixed total masses

Let $r > 0, m \in \mathbb{N}^*, \mu > 0, \delta > 0, j \in \mathbb{N}^*$.
By arguing as in ([5]), we can construct a function, $\chi^{r,m} \in C_0^\infty$ with range $[0,1]$ invariant under the collision transformations $J_{\alpha,\beta}$, defined for any $\{\alpha,\beta\} \in \{A,B\}$ by

$$J_{\alpha,\beta}(v, v_*, \omega) = (v^{(\alpha,\beta)\prime}, v_*^{(\alpha,\beta)\prime}, -\omega),$$

and under the exchange of v and v_*. Moreover $\chi^{r,m}$ satisfies also

$$\chi^{r,m}(v, v_*, \omega) = 1, \quad \forall (\alpha,\beta) \in \{A,B\}\ min(|\xi|, |\xi_*|, |\xi^{(\alpha,\beta)\prime}|, |\xi_*^{(\alpha,\beta)\prime}| \geq r),$$

and

$$\chi^{r,m}(v, v_*, \omega) = 0, \quad \forall (\alpha,\beta) \in \{A,B\}\ max(|\xi|, |\xi_*|, |\xi^{\alpha,\beta,\prime}|, |\xi_*^{\alpha,\beta,\prime}|) \leq r - \frac{1}{m}.$$

The modified collision kernel $\mathcal{B}^{\alpha,\beta}_{m,n,\mu}$ is a positive C^∞ function approximating $min(\mathcal{B}^{\alpha,\beta}, \mu)$, when

$$v^2 + v_*^2 < \frac{\sqrt{n}}{2}, \text{and } |\frac{v - v_*}{|v - v_*|} \cdot \omega| > \frac{1}{m}, \text{ and } |\frac{v - v_*}{|v - v_*|} \cdot \omega| < 1 - \frac{1}{m}$$

and such that $\mathcal{B}^{\alpha,\beta}_{m,n,\mu}(v, v_*, \omega) = 0$, if

$$v^2 + v_*^2 > \sqrt{n} \text{ or } |\frac{v - v_*}{|v - v_*|} \cdot \omega| < \frac{1}{2m}, \text{ or } |\frac{v - v_*}{|v - v_*|} \cdot \omega| > 1 - \frac{1}{2m}.$$

The functions φ_l are mollifiers in the x-variable defined by $\varphi_l(x) := l\varphi(lx)$, where

$$\varphi \in C_0^\infty(\mathbb{R}^3_v), \quad support(\varphi) \subset (-1,1), \quad \varphi \geq 0, \quad \int_{-1}^{1} \varphi(x)dx = 1.$$

For the sake of clarity Theorems 1.1 and 1.2 are shown for $M_A = M_B = 1$. The passage to general weighted masses M_A and M_B is immediate and we refer to ([6], [7], [11], [12]).

Non negative functions g_A, $g_B \in K$ and $\theta \in [0,1]$ are given. By arguing as in ([11]), we can construct F_A and F_B solutions of the following boundary value problem

$$\begin{aligned}
\delta F_A + \xi \frac{\partial}{\partial x} F_A = &\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^{r,m} \mathcal{B}^{AA}_{m,n,\mu} \frac{F_A}{1+\frac{F_A}{j}}(x,v') \frac{g_A * \varphi}{1+\frac{g_A*\varphi}{j}}(x,v_*^{'})dv_* d\omega \\
&+ \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^{r,m} \mathcal{B}^{AB}_{m,n,\mu} \frac{F_A}{1+\frac{F_A}{j}}(x,v') \frac{g_B * \varphi}{1+\frac{g_B*\varphi}{j}}(x,v_*^{'})dv_* d\omega \\
&- F_A \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^{r,m} \mathcal{B}^{AA}_{m,n,\mu} \frac{g_A * \varphi}{1+\frac{g_A*\varphi}{j}}(x,v_*)dv_* d\omega \\
- F_A \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} &\chi^{r,m} \mathcal{B}_{m,n,\mu} \frac{g_B * \varphi}{1+\frac{g_B*\varphi}{j}}(x,v_*)dv_* d\omega, \;\; (x,v) \in (-1,1)\times\mathbb{R}^3_v, \\
F_A(-1,v) = \lambda M_-(v),& \;\; \xi > 0, \quad F_A(1,v) = \lambda M_+(v), \;\; \xi < 0, \qquad (2.1)
\end{aligned}$$

and

$$\begin{aligned}
\delta F_B + \xi \frac{\partial}{\partial x} F_B = &\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^{r,m} \mathcal{B}^{BB}_{m,n,\mu} \frac{F_B}{1+\frac{F_B}{j}}(x,v') \frac{g_B * \varphi}{1+\frac{g_B*\varphi}{j}}(x,v_*^{'})dv_* d\omega \\
&+ \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^{r,m} \mathcal{B}^{AB}_{m,n,\mu} \frac{F_B}{1+\frac{F_B}{j}}(x,v') \frac{g_A * \varphi}{1+\frac{g_A*\varphi}{j}}(x,v_*^{'})dv_* d\omega \\
&- F_B \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^{r,m} \mathcal{B}^{BB}_{m,n,\mu} \frac{g_B * \varphi}{1+\frac{g_B*\varphi}{j}}(x,v_*)dv_* d\omega \\
- F_B \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} &\chi^{r,m} \mathcal{B}^{BA}_{m,n,\mu} \frac{g_A * \varphi}{1+\frac{g_A*\varphi}{j}}(x,v_*)dv_* d\omega, \;\; (x,v) \in (-1,1)\times\mathbb{R}^3_v, \\
F_B(-1,v) = \theta\lambda M_-(v),& \;\; \xi > 0, \quad F_B(1,v) = (1-\theta)\lambda M_+(v), \;\; \xi < 0, \quad (2.2)
\end{aligned}$$

as the L^1 limit of sequences. It can also been proven that the equations (2.1) and (2.2) each has a unique solution which is strictly positive. Hence the functions f_A and f_B,

$$f_A = \frac{F_A}{\int min(\mu, (1+|v|)^\beta) F_A(x,v) dx dv},$$
$$f_B = \frac{F_B}{\int min(\mu, (1+|v|)^\beta) F_B(x,v) dx dv}.$$

are well defined since F_A and F_B strictly positive.
Indeed using that $\int_{-1}^{1}(\alpha + \nu(x,v))dx \leq 2 + 2\mu$, it holds that

$$F_A(x,v) \geq \lambda M_-(v) e^{-\frac{2+2\mu}{\xi}}, \ \ \xi > 0, \qquad F_A(x,v) \geq \lambda M_+(v) e^{-\frac{2+2\mu}{|\xi|}}, \ \ \xi < 0.$$

Analogously, we obtain

$$F_B(x,v) \geq \theta \lambda M_-(v) e^{-\frac{2+2\mu}{\xi}}, \quad \xi > 0,$$
$$F_B(x,v) \geq (1-\theta) \lambda M_+(v) e^{-\frac{2+2\mu}{|\xi|}}, \quad \xi < 0.$$

By taking λ as

$$\lambda = min(\frac{1}{\int_{\xi>0} M_-(v) min(\mu, (1+|v|)^\beta) e^{-\frac{2+2\mu}{\xi}} dv}; \frac{1}{\int_{\xi<0} M_+(v) min(\mu, (1+|v|)^\beta) e^{-\frac{2+2\mu}{|\xi|}} dv}),$$

we get

$$\int min(\mu, (1+|v|)^\beta) F_A(x,v) dx dv \geq 1$$

and

$$\int min(\mu, (1+|v|)^\beta) F_B(x,v) dx dv \geq 1.$$

Hence the functions f_A and f_B are solutions to

$$
\begin{aligned}
\delta f_A+\xi\frac{\partial}{\partial x}f_A=&\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{AA}_{m,n,\mu}\frac{f_A}{1+\frac{F_A}{j}}(x,v')\frac{g_A*\varphi}{1+\frac{g_A*\varphi}{j}}(x,v'_*)dv_*d\omega\\
&+\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{AB}_{m,n,\mu}\frac{f_A}{1+\frac{F^A}{j}}(x,v')\frac{g_B*\varphi}{1+\frac{g_B*\varphi}{j}}(x,v'_*)dv_*d\omega\\
&-f_A\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{AA}_{m,n,\mu}\frac{g_A*\varphi}{1+\frac{g_A*\varphi}{j}}(x,v_*)dv_*d\omega\\
&-f_A\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{AB}_{m,n,\mu}\frac{g_B*\varphi}{1+\frac{g_B*\varphi}{j}}(x,v_*)dv_*d\omega,\quad (x,v)\in(-1,1)\times\mathbb{R}^3_v,\\
f_A(-1,v)=&\frac{\lambda}{\int min(\mu,(1+|v|)^\beta)F_A(x,v)dxdv}M_-(v),\ \ \xi>0,\\
f_A(1,v)=&\frac{\lambda}{\int min(\mu,(1+|v|)^\beta)F_A(x,v)dxdv}M_+(v),\ \ \xi<0,
\end{aligned}
\tag{2.3}
$$

and

$$
\begin{aligned}
\delta f_B+\xi\frac{\partial}{\partial x}f_B=&\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{BB}_{m,n,\mu}(v,v_*,\omega)\frac{f_B}{1+\frac{F_B}{j}}(x,v')\frac{g_B*\varphi}{1+\frac{g_B*\varphi}{j}}(x,v'_*)dv_*d\omega\\
&+\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{BA}_{m,n,\mu}(v,v_*,\omega)\frac{f_B}{1+\frac{F_B}{j}}(x,v')\frac{g_B*\varphi}{1+\frac{g_B*\varphi}{j}}(x,v'_*)dv_*d\omega\\
&-f_B(x,v)\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{BB}_{m,n,\mu}\frac{g_B*\varphi}{1+\frac{g_B*\varphi}{j}}(x,v_*)dv_*d\omega\\
&-f_B(x,v)\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{BA}_{m,n,\mu}\frac{g_A*\varphi}{1+\frac{g_A*\varphi}{j}}(x,v_*)dv_*d\omega,\ \ (x,v)\in(-1,1)\times\mathbb{R}^3_v,\\
f_B(-1,v)=&\frac{\lambda}{\int min(\mu,(1+|v|)^\beta)F_B(x,v)dxdv}\theta M_-(v),\quad \xi>0,\\
f_B(1,v)=&\frac{\lambda}{\int min(\mu,(1+|v|)^\beta)F_B(x,v)dxdv}(1-\theta)M_+(v),\quad \xi<0.
\end{aligned}
\tag{2.4}
$$

In order to use a fixed-point theorem, consider the closed and convex subset of $L^1_+([-1,1]\times\mathbb{R}^3_v)$,

$$
K=\{f\in L^1_+([-1,1]\times\mathbb{R}^3_v),\quad \int_{[-1,1]\times\mathbb{R}^3_v}min(\mu,(1+|v|)^\beta)f(x,v)dxdv=1\}.
$$

The fixed-point argument will now be used in order to solve (2.3, 2.4) with $g_A = f_A$ and $g_B = f_B$.

Define T on $K \times K \times [0,1]$ by $T(g_A, g_B, \theta) = (f_A, f_B, \tilde{\theta})$ with

$$\tilde{\theta} = \frac{\int_{\xi<0} |\xi| f_B(-1,v)dv}{\int_{\xi<0} |\xi| f_B(-1,v)dv + \int_{\xi>0} \xi f_B(1,v)dv} \tag{2.5}$$

and (f_A, f_B) solution to (2.3, 2.4).

The mapping T takes $K \times K \times [0,1]$ into itself. Next by using the exponetial forms of the equations (2.1, 2.2, 2.3, 2.4) together with averaging lemmas, it can be shown that the map T is continous and compact for the strong L^1 topology. So from the Schauder fixed point theorem there is (f_A, f_B, θ) such that

$$f_A = g_A, \quad f_B = g_B, \quad \theta = \frac{\int_{\xi<0} |\xi| f_B(-1,v)dv}{\int_{\xi>0} \xi f_B(1,v)dv + \int_{\xi<0} |\xi| f_B(-1,v)dv}$$

that satisfy

$$\begin{aligned}
\delta f_A + \xi \frac{\partial}{\partial x} f_A = &\int_{\mathbb{R}^3_{v_*} \times \mathbb{S}^2} \chi^{r,m} \mathcal{B}^{AA}_{m,n,\mu} \frac{f_A}{1+\frac{F_A}{j}}(x,v') \frac{f_A * \varphi_l}{1+\frac{f_A*\varphi_l}{j}}(x,v'_*)dv_*d\omega \\
&+ \int_{\mathbb{R}^3_{v_*} \times \mathbb{S}^2} \chi^{r,m} \mathcal{B}^{AB}_{m,n,\mu} \frac{f_A}{1+\frac{F_A}{j}}(x,v') \frac{f_B * \varphi_l}{1+\frac{f_B*\varphi_l}{j}}(x,v'_*)dv_*d\omega \\
&- f_A \int_{\mathbb{R}^3_{v_*} \times \mathbb{S}^2} \chi^{r,m} \mathcal{B}^{AA}_{m,n,\mu} \frac{f_A * \varphi_l}{1+\frac{f_A*\varphi_l}{j}}(x,v_*)dv_*d\omega \\
- f_A \int_{\mathbb{R}^3_{v_*} \times \mathbb{S}^2} &\chi^{r,m} \mathcal{B}^{AB}_{m,n,\mu} \frac{f_B * \varphi_l}{1+\frac{f_B*\varphi_l}{j}}(x,v_*)dv_*d\omega, \quad (x,v) \in (-1,1) \times \mathbb{R}^3_v, \\
f_A(-1,v) = k_A M_-(v), \quad &\xi > 0, \quad f_A(1,v) = k_A M_+(v), \quad \xi < 0
\end{aligned} \tag{2.6}$$

with

$$k_A = \frac{\lambda}{\int min(\mu, (1+|v|)^\beta) F_A(x,v)dxdv}$$

and

$$\delta f_B + \xi\frac{\partial}{\partial x} f_B = \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^{r,m}\mathcal{B}^{BB}_{m,n,\mu}\frac{f_B}{1+\frac{F^B}{j}}(x,v')\frac{f_B*\varphi_l}{1+\frac{f_B*\varphi_l}{j}}(x,v'_*)dv_*d\omega$$
$$+\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^{r,m}\mathcal{B}^{BA}_{m,n,\mu}\frac{f_B}{1+\frac{F_B}{j}}(x,v')\frac{f_A*\varphi_l}{1+\frac{f_A*\varphi_l}{j}}(x,v'_*)dv_*d\omega$$
$$-f_B\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^{r,m}\mathcal{B}^{BB}_{m,n,\mu}\frac{f_B*\varphi_l}{1+\frac{f_B*\varphi_l}{j}}(x,v_*)dv_*d\omega$$
$$-f_B\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^{r,m}\mathcal{B}^{BA}_{m,n,\mu}\frac{f_A*\varphi_l}{1+\frac{f_A*\varphi_l}{j}}(x,v_*)dv_*d\omega, \quad (x,v)\in(-1,1)\times\mathbb{R}^3_v,$$
$$f_B(-1,v)=\lambda'(\frac{\int_{\xi<0}|\xi|f_B(-1,v)dv}{\int_{\xi>0}\xi f_B(1,v)dv+\int_{\xi<0}|\xi|f_B(-1,v)dv})M_-(v), \;\; \xi>0,$$
$$f_B(1,v)=\lambda'(\frac{\int_{\xi>0}|\xi|f_B(1,v)dv}{\int_{\xi>0}\xi f_B(1,v)dv+\int_{\xi<0}|\xi|f_B(-1,v)dv})M_+(v), \;\; \xi<0, \tag{2.7}$$

with

$$\lambda' = \frac{\lambda}{\int min(\mu,(1+|v|)^\beta)F_B(x,v)dxdv}.$$

3 The slab solution for $-3<\beta\leq 0$ and $0\leq\beta<2$.

This section is devoted to the passage to the limit in (2.6, 2.7). It is performed in two steps. In the first one the solutions of the approached problem are written in their exponential form and averaging lemmas are used. The second passage to the limit corresponds to the passage to the limit in (3.8, 3.9). One crucial point is to get an entropy estimate on the sequence of approximations $(f^j_A, f^j_B)_{j\in\mathbb{N}}$ in order to extract compactness. In ([11]), this control is obtained from a bound on the entropy of $f^j = f^j_A + f^j_B$ by using that f^j satisfy the Boltzmann equation for a single component gas. But in the present paper, due to the difference of the molecular masses, this property is not satisfied.

Keeping, l, j, r, m, μ fixed, denote $f_A^{j,\delta,l,r,m,\mu}$ by f^δ_A and $f_B^{j,\delta,l,r,m,\mu}$ by f^δ_B. Writing the equations (2.6, 2.7) in the exponential form and using the averaging lemmas together with a convolution with a mollifier ([7],[19]) give that f^δ_A and F^δ_A are strongly compact in $L^1([-1,1]\times\mathbb{R}^3_v)$. Denote by f_A and F_A the respective limits of f^δ_A and F^δ_A. Following the proofs of ([6], [7], [11]) a strong compactness argument is used to pass to the limit in (2.6) when δ tends to 0. Hence f_A is solution to

$$\xi\frac{\partial}{\partial x}f_A = \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{AA}_{m,n,\mu}\frac{f_A}{1+\frac{F^A}{j}}(x,v')\frac{f_A*\varphi_l}{1+\frac{f_A*\varphi_l}{j}}(x,v_*^{'})dv_*d\omega$$
$$\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{AB}_{m,n,\mu}\frac{f_A}{1+\frac{F^A}{j}}(x,v')\frac{f_B*\varphi_l}{1+\frac{f_B*\varphi_l}{j}}(x,v_*^{'})dv_*d\omega$$
$$-f_A\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{AA}_{m,n,\mu}\frac{f_A*\varphi_l}{1+\frac{f_A*\varphi_l}{j}}(x,v_*)dv_*d\omega,$$
$$-f_A\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{AB}_{m,n,\mu}\frac{f_B*\varphi_l}{1+\frac{f_B*\varphi_l}{j}}(x,v_*)dv_*d\omega,\quad (x,v)\in(-1,1)\times\mathbb{R}^3_v,$$
$$f_A(-1,v) = \frac{\lambda}{\int min(\mu,(1+|v|)^\beta)F_A(x,v)dxdv}M_-(v),\quad \xi>0,$$
$$f_A(1,v) = \frac{\lambda}{\int min(\mu,(1+|v|)^\beta)F_A(x,v)dxdv}M_+(v),\quad \xi<0, \tag{3.8}$$

with

$$\int min(\mu,(1+|v|)^\beta)f_A^j(x,v)dxdv = 1.$$

For the same reasons, the limit f_B of f_B^δ satisfies

$$\xi\frac{\partial}{\partial x}f_B = \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{BB}_{m,n,\mu}\frac{f_B}{1+\frac{F^B}{j}}(x,v')\frac{f_B*\varphi_l}{1+\frac{f_B*\varphi_l}{j}}(x,v_*^{'})dv_*d\omega$$
$$+\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{BB}_{m,n,\mu}\frac{f_B}{1+\frac{F^B}{j}}(x,v')\frac{f_A*\varphi_l}{1+\frac{f_A*\varphi_l}{j}}(x,v_*^{'})dv_*d\omega$$
$$-f_B\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{BB}_{m,n,\mu}\frac{f_B*\varphi_l}{1+\frac{f_B*\varphi_l}{j}}(x,v_*)dv_*d\omega$$
$$-f_B\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}^{BA}_{m,n,\mu}\frac{f_A*\varphi_l}{1+\frac{f_A*\varphi_l}{j}}(x,v_*)dv_*d\omega,\quad (x,v)\in(-1,1)\times\mathbb{R}^3_v,$$
$$f_B(-1,v) = \sigma(-1)\lambda' M_-(v),\ \ \xi>0,\quad f_B(1,v)=\sigma(1)\lambda' M_+(v),\ \ \xi<0, \tag{3.9}$$

with

$$\int min(\mu,(1+|v|)^\beta)f_B(x,v)dxdv = 1,$$

where

$$\sigma(-1) = \frac{\int_{\xi<0} |\xi| f_B(-1,v)dv}{\int_{\xi>0} \xi f_B(1,v)dv + \int_{\xi<0} |\xi| f_B(-1,v)dv},$$
$$\sigma^j(1) = \frac{\int_{\xi>0} \xi f_B(1,v)dv}{\int_{\xi>0} \xi f_B(1,v)dv + \int_{\xi<0} |\xi| f_B(-1,v)dv}$$

and

$$\lambda' = \frac{\lambda}{\int min(\mu, (1+|v|)^{\beta}) F_B^j(x,v)dxdv}.$$

Mutltiply (3.8) by $\log(\frac{f_A^j}{1+\frac{f_A^j}{j}})$ and (3.9) by $\log(\frac{f_B^j}{1+\frac{f_B^j}{j}})$ and add the two resulting equations leads to according to ([6], [2], [17]),

$$\int_{\mathbb{R}^3} \xi \left(f_A^j \log(f_A^j)(1,v) - j(1+\frac{f_A^j}{j})\log(1+\frac{f_A^j}{j})(1,v) \right)$$

$$-\int_{\mathbb{R}^3} \xi \left(f_A^j \log(f_A^j)(-1,v) - j(1+\frac{f_A^j}{j})\log(1+\frac{f_A^j}{j})(-1,v) \right)$$

$$+\int_{\mathbb{R}^3} \xi \left(f_B^j \log(f_B^j)(1,v) - j(1+\frac{f_B^j}{j})\log(1+\frac{f_B^j}{j})(1,v) \right)$$

$$-\int_{\mathbb{R}^3} \xi \left(f_B^j \log(f_B^j)(1,v) - j(1+\frac{f_B^j}{j})\log(1+\frac{f_B^j}{j})(1,v) \right)$$

$$= -\frac{1}{4} I^j_{AA}(f_A^j, f_A^j) - \frac{1}{2} I^j_{AB}(f_A^j, f_B^j) - \frac{1}{4} I^j_{BB}(f_B^j, f_B^j)$$

$$+\int \chi^{r,m} \mathcal{B}^{AA}_{m,n,\mu} \frac{f_A^{j\prime}(f_A^{j\prime} - F_A^{j\prime})}{j(1+F_A^{j\prime})(1+f_A^{j\prime})} \frac{f'_{A*}}{1+\frac{f^{j\prime}_{A*}}{j}} \log \frac{f_A^j}{1+\frac{f_A^j}{j}}$$

$$+\int \chi^{r,m} \mathcal{B}^{AB}_{m,n,\mu} \frac{f_A^{j\prime}(f_A^{j\prime} - F_A^{j\prime})}{j(1+F_A^{j\prime})(1+f_A^{j\prime})} \frac{f^{j\prime}_{B*}}{1+\frac{f'_{B*}}{j}} \log \frac{f_A^j}{1+\frac{f_A^j}{j}}$$

$$-\int \chi^{r,m} \frac{f_A^{j2}}{j(1+\frac{f_A^j}{j})} \log \frac{f_A^j}{1+\frac{f_A^j}{j}} \left(\mathcal{B}^{AA}_{m,n,\mu} \frac{f^j_{A*}}{(1+\frac{f^j_{A*}}{j})} + \mathcal{B}^{AB}_{m,n,\mu} \frac{f^j_{B*}}{(1+\frac{f^j_{B*}}{j})} \right)$$

$$+\int \chi^{r,m} \mathcal{B}^{BB}_{m,n,\mu} \frac{f_B^{j\prime}(f_B^{j\prime} - F_B^{j\prime})}{j(1+F_B^{j\prime})(1+f_B^{j\prime})} \frac{f^{j\prime}_{B*}}{1+\frac{f'_{B*}}{j}} \log \frac{f_B^j}{1+\frac{f_B^j}{j}}$$

$$+\int \chi^{r,m} \mathcal{B}^{BA}_{m,n,\mu} \frac{f'_B(f'_B - F'_B)}{j(1+F'_B)(1+f'_B)} \frac{f'_{A*}}{1+\frac{f'_{A*}}{j}} \log \frac{f_B}{1+\frac{f_B}{j}}$$

$$-\int \chi^{r,m} \frac{f_B^{j2}}{j(1+\frac{f_B^j}{j})} \log \frac{f_B^j}{1+\frac{f_B^j}{j}} \left(\mathcal{B}^{BB}_{m,n,\mu} \frac{f^j_{B*}}{(1+\frac{f^j_{B*}}{j})} + \mathcal{B}^{BA}_{m,n,\mu} \frac{f^j_{A*}}{(1+\frac{f^j_{A*}}{j})} \right)$$

with

$$I^j_{AA}(f^j_A, f^j_A) = \int \chi^{r,m} \mathcal{B}^{AA}_{m,n,\mu} \left(\frac{f^{j\prime}_A}{1+\frac{f^{j\prime}_A}{j}} \frac{f^{j\prime}_{A*}}{1+\frac{f^{j\prime}_{A*}}{j}} - \frac{f^j_A}{1+\frac{f^j_A}{j}} \frac{f^j_{A*}}{1+\frac{f^j_{A*}}{j}} \right) \log \left(\frac{\frac{f'_A}{1+\frac{f'_A}{j}} \frac{f^{j\prime}_{A*}}{1+\frac{f^{j\prime}_{A*}}{j}}}{\frac{f^j_A}{1+\frac{f^j_A}{j}} \frac{f^j_{A*}}{1+\frac{f^j_{A*}}{j}}} \right) dx dv dv_* d\omega,$$

$$I^j_{BB}(f^j_B, f^j_B) = \int \chi^{r,m} \mathcal{B}^{BB}_{m,n,\mu} \left(\frac{f'_B}{1+\frac{f'_B}{j}} \frac{f'_{B*}}{1+\frac{f'_{B*}}{j}} - \frac{f^j_B}{1+\frac{f^j_B}{j}} \frac{f^j_{B*}}{1+\frac{f^j_{B*}}{j}} \right) \log \left(\frac{\frac{f^{j\prime}_B}{1+\frac{f^{j\prime}_B}{j}} \frac{f^{j\prime}_{B*}}{1+\frac{f^{j\prime}_{B*}}{j}}}{\frac{f^j_B}{1+\frac{f^j_B}{j}} \frac{f^j_{B*}}{1+\frac{f^j_{B*}}{j}}} \right) dx dv dv_* d\omega,$$

$$I^j_{AB}(f^j_A, f^j_B) = \int \chi^{r,m} \mathcal{B}^{AB}_{m,n,\mu} \left(\frac{f^{j\prime}_A}{1+\frac{f^{j\prime}_A}{j}} \frac{f^{j\prime}_{B*}}{1+\frac{f^{j\prime}_{B*}}{j}} - \frac{f^j_A}{1+\frac{f^j_A}{j}} \frac{f^j_{B*}}{1+\frac{f^j_{B*}}{j}} \right) \log \left(\frac{\frac{f^{j\prime}_A}{1+\frac{f^{j\prime}_A}{j}} \frac{f^{j\prime}_{B*}}{1+\frac{f^{j\prime}_{B*}}{j}}}{\frac{f_A}{1+\frac{f^j_A}{j}} \frac{f^j_{B*}}{1+\frac{f^j_{B*}}{j}}} \right) dx dv dv_* d\omega.$$

From ([2]), we have $I^j_{AA}(f^j_A, f^j_A) \geq 0$, $I^j_{AB}(f^j_A, f^j_B) \geq 0$ $I^j_{BB}(f^j_B, f^j_B) \geq 0$. Moreover by reasonning as in ([6]), it can proved that the terms

$$-\int \chi^{r,m} \mathcal{B}^{\alpha\beta}_{m,n,\mu} \frac{f^2_\alpha}{j(1+\frac{f_\alpha}{j})} \frac{f_{\beta*}}{(1+\frac{f_{\beta*}}{j})} \log \frac{f_\alpha}{1+\frac{f_\alpha}{j}}, \tag{3.10}$$

$$\int \chi^{r,m} \mathcal{B}^{\alpha,\beta}_{m,n,\mu} \frac{f'_\alpha(f'_\alpha - F'_\alpha)}{j(1+F'_\alpha)(1+f'_\alpha)} \frac{f'_{\beta*}}{1+\frac{f'_{\beta*}}{j}} \log \frac{f_\alpha}{1+\frac{f_\alpha}{j}} \tag{3.11}$$

are bounded uniformly in j. For the sake of clarity the proof of the control of the terms (3.10, 3.11) are written in the appendix. Therefore

$$\int_{\mathbb{R}^3}\xi\left(f_A^j\log(f_A^j)(1,v)-j(1+\frac{f_A^j}{j})\log(1+\frac{f_A^j}{j})(1,v)\right)$$
$$-\int_{\mathbb{R}^3}\xi\left(f_A^j\log(f_A^j)(-1,v)-j(1+\frac{f_A^j}{j})\log(1+\frac{f_A^j}{j})(-1,v)\right)$$
$$+\int_{\mathbb{R}^3}\xi\left(f_B^j\log(f_B^j)(1,v)-j(1+\frac{f_B^j}{j})\log(1+\frac{f_B^j}{j})(1,v)\right)$$
$$-\int_{\mathbb{R}^3}\xi\left(f_B^j\log(f_B^j)(-1,v)-j(1+\frac{f_B^j}{j})\log(1+\frac{f_B^j}{j})(-1,v)\right)\leq c$$

So by arguing as in ([6], [7]), the entropies of f_A^j and f_B^j can be bounded uniformly in j. Hence f_A^j and f_B^j are weakly compact in L^1.

Remark 1. *Contrarily to ([11], [12]), the weak compactness of f_A^j and f_B^j is directly obtained. In ([11], [12]), the author shows that the sum $f^j = f_A^j + f_B^j$ is weakly compact in L^1 by using that f^j satisfies the Boltzmann equation for a single component gas. In the present paper, the 2 components having different molecular masses, f^j is not solution of the Boltzmann equation for a one component gas.*

Remark 2. *The quantity $\frac{1}{4}I_{AA}^j(f_A^j, f_A^j) + \frac{1}{2}I_{AB}^j(f_A^j, f_B^j) + \frac{1}{4}I_{BB}^j(f_B^j, f_B^j)$ is a generalization of the entropy production term used in ([6]).*

Let $Q_{\alpha,\beta}^{j-}$ and $Q_{\alpha,\beta}^{j+}$ be defined by

$$Q_{\alpha,\beta}^{j-}(f_\alpha^j, f_\beta^j) = f_\alpha^j(x,v)\int_{\mathbb{R}^3\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}_{m,n,\mu}\frac{f_\beta^j}{1+\frac{f_\beta^j}{j}}(x,v_*)dv_*d\omega,$$
$$Q_{\alpha,\beta}^{j+}(f_\alpha^j, f_\beta^j) = \int_{\mathbb{R}^3\times\mathbb{S}^2}\chi^{r,m}\mathcal{B}_{m,n,\mu}\frac{f_\alpha^j}{1+\frac{f_\alpha^j}{j}}(x,v')\frac{f_\beta^j}{1+\frac{f_\beta^j}{j}}(x,v'_*)dv_*d\omega.$$

In order to pass to the limit in (3.8, 3.9) weak compactness is required on the terms $Q_{\alpha,\beta}^{j-}$ and $Q_{\alpha,\beta}^{j+}$. For any $\{\alpha,\beta\}\in\{A,B\}$, the inequalities

$$Q_{\alpha,\beta}^{j-}(f_\alpha^j, f_\beta^j)\leq c\, f_\alpha^j,$$

with c independant of j, give that $Q^{j-}_{\alpha,\beta}$ is weakly compact in L^1. By arguing as in a one component gas, we can show that

$$Q^{j+}_{A,A}(f^j_A, f^j_A) + Q^{j+}_{A,B}(f^j_A, f^j_B) \leq K\left(Q^{j-}_{A,A}(f^j_A, f^j_A) + Q^{j-}_{A,B}(f^j_A, f^j_B)\right)$$
$$+\frac{1}{\ln K}\left(I_{AA}(f^j_A, f^j_A) + \int \left(f_A(x,v')f_B(x,v'_*) - f_A(x,v)f_B(x,v_*)\ln\left(\frac{f_A(x,v)}{f_A(x,v')}\right)\right)\right). \quad (3.12)$$

and

$$Q^{j+}_{B,A}(f^j_B, f^j_A) + Q^{j+}_{B,B}(f^j_B, f^j_B) \leq K\left(Q^{j-}_{B,B}(f^j_B, f^j_B) + Q^{j-}_{B,A}(f^j_B, f^j_A)\right)$$
$$+\frac{1}{\ln K}\left(I_{AA}(f^j_B, f^j_B) + \int \left(f_A(x,v')f_B(x,v'_*) - f_A(x,v)f_B(x,v_*)\ln\left(\frac{f_B(x,v)}{f_B(x,v')}\right)\right)\right). \quad (3.13)$$

By adding the two inequalities (3.10, 3.11), we get

$$Q^{j+}_{A,A}(f^j_A, f^j_A) + Q^{j+}_{A,B}(f^j_A, f^j_B) + Q^{j+}_{B,A}(f^j_B, f^j_A) + Q^{j+}_{B,B}(f^j_B, f^j_B)$$
$$\leq K\left(Q^{j-}_{A,A}(f^j_A, f^j_A) + Q^{j-}_{A,B}(f^j_A, f^j_B) + Q^{j-}_{B,A}(f^j_B, f^j_A) + Q^{j-}_{B,B}(f^j_B, f^j_B)\right)$$
$$\frac{1}{\ln(K)}\left(I_{AA}(f^j_A, f^j_A) + I_{BB}(f^j_B, f^j_B) + I_{BA}(f^j_B, f^j_A)\right).$$

From the weak compactness of $Q^{j-}_{\alpha,\beta}$ for $\{\alpha,\beta\} \in \{A,B\}$ and the boundeness from above of

$$I_{AA}(f^j_A, f^j_A) + I_{BB}(f^j_B, f^j_B) + I_{BA}(f^j_B, f^j_A),$$

the gain terms $Q^{j+}_{\alpha,\beta}$ are weakly compact in L^1 for any $\{\alpha,\beta\} \in \{A,B\}$. Hence by arguing as in ([6], [7]) we can pass to the limit in the equations (3.8, 3.9). So there is $(f^{r,\mu}_A, f^{r,\mu}_B)$ solution to

$$\xi\frac{\partial}{\partial x}f^{r,\mu}_A = \int_{\mathbb{R}^3_v\times\mathbb{S}^2}\chi^r\mathcal{B}^{AA}_\mu(v-v_*,\omega)f^{r,\mu}_A(x,v')f^{r,\mu}_A(x,v'_*)dv_*d\omega$$
$$+\int_{\mathbb{R}^3_v\times\mathbb{S}^2}\chi^r\mathcal{B}^{AB}_\mu(v-v_*,\omega)f^{r,\mu}_A(x,v')f^{r,\mu}_B(x,v'_*)dv_*d\omega$$
$$-f^{r,\mu}_A\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^r\mathcal{B}_\mu(v-v_*,\omega)f^{r,\mu}_A(x,v_*)dv_*d\omega,$$
$$-f^{r,\mu}_A\int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2}\chi^r\mathcal{B}^{AB}_\mu(v-v_*,\omega)f^{r,\mu}_B(x,v_*)dv_*d\omega, \quad (x,v)\in(-1,1)\times\mathbb{R}^3_v,$$
$$f^{r,\mu}_A(-1,v) = k_AM_-(v),\ \xi>0, \quad f^{r,\mu}_A(1,v) = k_AM_+(v),\ \xi<0, \quad (3.14)$$

with

$$\int min(\mu, (1+|v|)^{\beta}) f_A^{r,\mu}(x,v)dxdv = 1,$$

where k_A is defined in the equation (2.6) before passing to the limit.

$$\xi \frac{\partial}{\partial x} f_B^{r,\mu} = \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^r \mathcal{B}_\mu^{BB}(v-v_*,\omega) f_B^{r,\mu}(x,v') f_B^{r,\mu}(x,v_*^{'}) dv_* d\omega$$
$$+ \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^r \mathcal{B}_\mu^{AB}(v-v_*,\omega) f_A^{r,\mu}(x,v') f_B^{r,\mu}(x,v_*^{'}) dv_* d\omega$$
$$-f_B^{r,\mu} \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^r \mathcal{B}_\mu^{BB}(v-v_*,\omega) f_B^{r,\mu}(x,v_*) dv_* d\omega$$
$$-f_B^{r,\mu} \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^r \mathcal{B}_\mu^{BA}(v-v_*,\omega) f_A^{r,\mu}(x,v_*) dv_* d\omega, \quad (x,v) \in (-1,1)\times\mathbb{R}^3_v,$$
$$f_B^{r,\mu}(-1,v) = \sigma(-1)\lambda' M_-(v),\ \xi>0, \quad f_B^{r,\mu}(1,v) = \sigma(1)\lambda' M_+(v),\ \xi<0, \tag{3.15}$$

with

$$\int min(\mu, (1+|v|)^{\beta}) f_B^{r,\mu}(x,v)dxdv = 1.$$

Here, $\sigma(-1)$ and $\sigma(1)$ have the expressions

$$\sigma(-1) = \frac{\int_{\xi<0} |\xi| f_B^{r,\mu}(-1,v)dv}{\int_{\xi>0} \xi f_B^{r,\mu}(1,v)dv + \int_{\xi<0} |\xi| f_B^{r,\mu}(-1,v)dv}$$

and

$$\sigma(1) = \frac{\int_{\xi>0} \xi f_B^{r,\mu}(1,v)dv}{\int_{\xi>0} \xi f_B^{r,\mu}(1,v)dv + \int_{\xi<0} |\xi| f_B^{r,\mu}(-1,v)dv}.$$

By using the mass conservation as in ([11]), the boundary conditions of (3.15) writes

$$f_B^{r,\mu}(-1,v) = M_-(v) \int_{\xi<0} |\xi| f_B^{r,\mu}(-1,v)dv,\ \ \xi>0,$$
$$f_B^{r,\mu}(1,v) = M_+(v) \int_{\xi>0} \xi f_B^{r,\mu}(1,v)dv,\ \ \xi<0. \tag{3.16}$$

Let $(r_j)_{j\in\mathbb{N}}$ with $r_j \to 0$ and μ_j with $\mu_j \to +\infty$, $f_A^j = f_A^{r_j,\mu_j}$ and $f_B^j = f_B^{r_j,\mu_j}$. Next we pass to the limit in the weak formulations satisfied by f_A^j and f_B^j for $0 \leq \beta < 2$. By using averaging lemmas as in ([6], [7], [11] [12]), we get

$$\lim_{j\to+\infty} \int Q^{j-}_{\alpha,\beta}(f^j_\alpha, f^j_\beta)\,\varphi\,dxdv = \int Q^-_{\alpha,\beta}(f_\alpha, f_\beta)\,\varphi\,dxdv.$$

Moreover by using th change of variable $(v, v_*, \omega) \to (v', v'_*, -\omega)$, the same result holds for the gain terms

$$\lim_{j\to+\infty} \int Q^{j+}_{\alpha,\beta}(f^j_\alpha, f^j_\beta)\,\varphi\,dxdv = \int Q^+_{\alpha,\beta}(f_\alpha, f_\beta)\,\varphi\,dxdv.$$

Finally (f_A, f_B) satisfies (1.1, 1.2) in the weak sense for $0 \leq \beta < 2$. In the situation where $-3 < \beta \leq 0$ the passage to the limit is realized in the weak reformulation.

But for the sake of clarity we explain the passage to the limit in the terms (3.16) i.e we prove the weak convergence in $L^1(\{v \in \mathbb{R}^3_v, \xi > 0\})$ (resp $L^1(\{v \in \mathbb{R}^3_v, \xi < 0\})$) of $f^j_B(1,.)$ (resp. $f^j_B(-1,.)$) to $f_B(1,.)$ (resp. $f_B(-1,.)$). First, it is important to check that the fluxes $\int_{\xi>0} \xi f^j_B(1,v)dv$ and $\int_{\xi<0} |\xi| f^j_B(-1,v)dv$ are controled. From (3.15) written in the exponential form, it holds that

$$\begin{aligned} & f^j_B(x,v) \geq \\ & f^j_B(-1,v) e^{-\int_{-\frac{1+x}{\xi}}^0 \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^r(\mathcal{B}^\mu_{BA} f^{r,\mu}_A(x+s\xi,v_*) + \mathcal{B}^\mu_{BB} f^{r,\mu}_B(x+s\xi,v_*)dv_* d\omega ds}, \\ & \qquad \xi > \frac{1}{2}, |v| \leq 2, \\ & f^j_B(x,v) \geq \\ & f^j_B(1,v) e^{-\int_{\frac{1-x}{\xi}}^0 \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \chi^r(\mathcal{B}^\mu_{BA} f^j_A(x+s\xi,v_*)dv_* + \mathcal{B}^\mu_{BB} f^j_B(x+s\xi,v_*)dv_* d\omega ds}, \\ & \qquad \xi < -\frac{1}{2}, |v| \leq 2. \end{aligned} \tag{3.17}$$

For v satisfying $|v| \leq 2$ with $\xi > \frac{1}{2}$ or $\xi < -\frac{1}{2}$,

$$\int_{-1}^1 \int_{\mathbb{R}^3_{v_*}\times\mathbb{S}^2} \frac{\chi^r}{|\xi|} \left(\mathcal{B}^\mu_{BA} f^{r,\mu}_A(z,v) + \mathcal{B}^\mu_{BB} f^{r,\mu}_B(z,v)\right) dv_* d\omega dz$$

is uniformly bounded from above. Hence, using the definition of the boundary conditions (1.6) in (3.17), it comes

$$\begin{aligned} f^j_B(x,v) &\geq cM_-(v)\int_{\xi<0}|\xi|f^j_B(-1,v)dv, \quad \xi>\frac{1}{2}, \quad |v|\leq 2,\\ f^j_B(x,v) &\geq cM_+(v)\int_{\xi>0}\xi f^j_B(1,v)dv, \quad \xi<-\frac{1}{2}, \quad |v|\leq 2. \end{aligned}$$

So,

$$\begin{aligned} &c\int_{\{\xi>\frac{1}{2},|v|\leq 2\}\cup\{\xi<-\frac{1}{2},|v|\leq 2\}} f^j_B(x,v)dxdv\\ &\geq \int_{\xi>0}\xi f^j_B(1,v)dv+\int_{\xi<0}|\xi|f^j_B(-1,v)dv. \end{aligned}$$

f^j_B being non negative,

$$\begin{aligned} &c\int_{-1}^1\int_{\mathbb{R}^3_v} min(\mu,(1+|v|)^\beta)f^j_B(x,v)dxdv\\ &\geq \int_{\xi>0}\xi f^j_B(1,v)dv+\int_{\xi<0}|\xi|f^j_B(-1,v)dv. \end{aligned}$$

Since $\int_{-1}^1\int_{\mathbb{R}^3_v} min(\mu,(1+|v|)^\beta)f^j_B(x,v)dxdv=1$, the fluxes $\int_{\xi>0}\xi f^j_B(1,v)dv$ and $\int_{\xi<0}|\xi|f^j_B(-1,v)dv$ are bounded uniformly w.r.t j.
Furthermore, the energy fluxes are also controlled. Indeed, from Property 1.1, the conservation of energy for (f^j_A,f^j_B) gives

$$\begin{aligned} &m^B\left(\int_{\xi>0}\xi v^2 f^j_B(1,v)dv+\int_{\xi<0}|\xi|v^2 f^j_B(-1,v)dv\right)\\ &\leq \int_{\xi>0}\xi v^2(m_A f^j_A(-1,v)+m_B f^j_B(-1,v))dv\\ &+\int_{\xi<0}|\xi|v^2(m_A f^j_A(1,v)+m_B f^j_A(1,v))dv. \end{aligned}$$

By definition of the boundary conditions (3.14) and (3.15),

$$\begin{aligned} &\int_{\xi>0}\xi v^2 f^j_B(1,v)dv+\int_{\xi<0}|\xi|v^2 f^j_B(-1,v)dv\\ &\leq (\frac{m^A}{m^B}k^j+\int_{\xi'<0}|\xi'|f^j_B(-1,v')dv')\int_{\xi>0}\xi v^2 M_-(v)dv\\ &+(\frac{m^A}{m^B}k^j+\int_{\xi'>0}\xi' f^j_B(1,v')dv')\int_{\xi<0}|\xi|v^2 M_+(v)dv. \end{aligned} \quad (3.18)$$

The right-hand side of (3.18) being bounded, the energy fluxes are also bounded. Finally, the entropy fluxes can also be controled. Indeed

$$\xi\frac{\partial}{\partial x}\big(f_A^j(log(f_A^j)-1)\big)=Q_{AA}^j(f_A^j,f_A^j)log(f_A^j)+Q_{AB}^j(f_A^j,f_B^j)log(f_A^j),$$
$$\xi\frac{\partial}{\partial x}\big(f_B^j(log(f_B^j)-1)\big)=Q_{BB}^j(f_B^j,f_B^j)\log(f_B^j)+Q_{BA}^j(f_B^j,f_A^j)\log(f_B^j). \tag{3.19}$$

Using a Green's formula and an entropy estimate in the system (3.19), leads to

$$\begin{aligned}\int_{\xi>0}\xi f_B^j(1,v)\log f_B^j(1,v)dv+\int_{\xi<0}|\xi|f_B^j(-1,v)\log f_B^j(-1,v)dv\\ \leq(\int_{\xi'>0}\xi' f_B^j(1,v')dv'+k^j)\\ \int_{\xi<0}|\xi|M_+(v)\log(M_+(v)(\int_{\xi'>0}\xi' f_B^j(1,v')dv'+k^j))dv\\ +(\int_{\xi'<0}|\xi'|f_B^j(-1,v')dv'+k^j)\\ \int_{\xi>0}M_-(v)\log(M_-(v)(\int_{\xi'<0}|\xi'|f_B^j(-1,v')dv'+k^j))dv.\end{aligned}$$

By the Dunford-Pettis criterion ([14]), $f_B^j(1,.)$ is weakly compact in $L^1(\{v\in\mathbb{R}_v^3,\xi>0\})$. Let one of its subsequence still denoted by $f_B^j(1,.)$, converging weakly to some g_+ in $L^1(\{v\in\mathbb{R}_v^3,\xi>0\})$. Next the aim is to identify g_+ and $f_B(1,v)$. We recall that the trace $f_B(1,v)$ can be defined by

$$f_B(1,v)=\lim_{\epsilon_0\to 0}\frac{1}{\epsilon_0}\int_0^{\epsilon_0}f_B(1-\epsilon,v)d\epsilon\quad([10]).$$

$(\varphi f_B^j)_{j\in\mathbb{N}}$ satisfies

$$\xi\frac{\partial(\varphi f_B^j)}{\partial x}=\xi\frac{\partial\varphi}{\partial x}f_B^j+Q_j(f_B^j,f^j)\varphi. \tag{3.20}$$

So by integrating 3.20 on $[1-\varepsilon,1]\times\mathbb{R}^3$ and by using a Green's formula, it holds that

$$|\frac{1}{\epsilon_0}\int_{\mathbb{R}^3_v}\int_0^{\epsilon_0}(f^j_B(1,v)-f^j_B(1-\epsilon,v))\varphi_2(v)dvd\epsilon|$$

$$\leq \frac{1}{\epsilon_0}\int_0^{\epsilon_0}\int_{\mathbb{R}^3_v}\int_{1-\epsilon_0}^1 |Q_j(f^j_B,f^j)(x,v)\varphi(x,v)|dxdvd\epsilon$$

$$+\frac{1}{\epsilon_0}\int_0^{\epsilon_0}\int_{\mathbb{R}^3_v}\int_{1-\epsilon_0}^1 |f^j_B(x,v)\xi\frac{\partial}{\partial x}\varphi(x,v)|dxdvd\epsilon. \tag{3.21}$$

Hence by using the weak compactess of f^j_B and $Q_j(f^j_B,f^j)$ and by passing to the limit in (3.21), g_+ and $f_B(1,v)$ can be identified. This concludes the proof of Theorems 1 and 2.

Appendix: Proofs of (3.10, 3.11)

$$-\int \chi^{r,m}\mathcal{B}^{\alpha\beta}_{m,n,\mu}\frac{f^2_\alpha}{j(1+\frac{f_\alpha}{j})}\frac{f_{\beta *}}{(1+\frac{f_{\beta *}}{j})}\log\frac{f_\alpha}{1+\frac{f_\alpha}{j}}$$

$$\leq -\int_{\frac{f_\alpha}{1+\frac{f_\alpha}{j}}<1}\chi^{r,m}\mathcal{B}^{\alpha\beta}_{m,n,\mu}\frac{f^2_\alpha}{j(1+\frac{f_\alpha}{j})}\frac{f_{\beta *}}{(1+\frac{f_{\beta *}}{j})}\log\frac{f_\alpha}{1+\frac{f_\alpha}{j}},$$

But for any $x\in]0,1]$, $-x\log(x)\leq \frac{2}{e}$, it holds that

$$-\int \chi^{r,m}\mathcal{B}^{\alpha\beta}_{m,n,\mu}\frac{f^2_\alpha}{j(1+\frac{f_\alpha}{j})}\frac{f_{\beta *}}{(1+\frac{f_{\beta *}}{j})}\log\frac{f_\alpha}{1+\frac{f_\alpha}{j}}$$

$$\leq -\frac{2}{e}\int_{\frac{f_\alpha}{1+\frac{f_\alpha}{j}}<1}\chi^{r,m}\mathcal{B}^{\alpha\beta}_{m,n,\mu}\frac{f_\alpha}{j}\frac{f_{\beta *}}{(1+\frac{f_{\beta *}}{j})}$$

Hence f_α and f_β having M_α and M_β for weighted masses

$$-\int \chi^{r,m}\mathcal{B}^{\alpha\beta}_{m,n,\mu}\frac{f^2_\alpha}{j(1+\frac{f_\alpha}{j})}\frac{f_{\beta *}}{(1+\frac{f_{\beta *}}{j})}\log\frac{f_\alpha}{1+\frac{f_\alpha}{j}}\leq cM_\alpha M_\beta$$

and (3.10) follows. The proof of (3.11) is analogous.

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