Numerical methods for the non-conservative bi-temperature Euler model

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The aim of this work is the study of out-of-equilibirum plasma physics. It is a multiscale problem involving both very small lengths (Debye length) and high frequency oscillations (electronic plasma frequency). Transport of charged particles (electrons and ions) in context of Inertial Confinement Fusion (ICF) can be modelled by the bi-temperature Euler equations, which are a non-conservative hyperbolic system. It contains so-called non-conservative terms, which cannot be put in divergential form. Such terms are not well-defined, and, in situations involving shocks, computing exact or approximated solutions is a challenging issue.

The bi-temperature Euler model can be recovered by using a Chapman-Enskog expansion on an underlying kinetic approach of this system, the Vlasov-BGK-Ampère system, which is conservative. We are interested in the numerical resolution of this kinetic model, in a macroscopic setting. Hence, a scaling is performed on this model in order to exhibit the behaviour of the system in large scale configurations. The major issue of such a system is that the Maxwell equations are describing small scale electromagnetics. At the macroscopic level, these equations degenerate into algebraic relations, preventing their use for computation purposes. Hence, we derive an Asymptotic-Preserving numerical method, which is able to solve the system even when these small scales (Debye length, electronic plasma frequency) are not resolved, i.e Δt, $\Delta x \leq \varepsilon$, with $\varepsilon \to 0$.

Secondly, a Suliciu relaxation method for the Bi-temperature Euler system is derived in order to provide comparaisons with the first method. Several test cases are studied. They consist in well-known Riemann problems that are solved with both methods in order to exhibit the behaviour of the non-conservative terms through shocks.