Uniform derivation of Coulomb collisional transport thanks to Debye shielding

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Outline

1. Debye shielding, Landau damping... and transport
2. Periodic jellium model
3. Linearizing near ballistic approximation
   - dynamics
   - effective potential
4. Perturbative approximation (Picard)
   - Close collisions – strong deflections
   - Small deflections by distant particles
5. Vlasov limit : smoothing ($N \to \infty$)
   - from Coulomb to effective potential
   - field evolution
6. Final comments

Debye shielding, Landau damping and velocity diffusion

Debye shielding :

- how do near-ballistic, thermalized particles shield each other ?
- statistical formulation from equilibrium pair correlations, after BBGKY expansion

Landau damping :

- what are particles doing ? (see Villani-Mouhot)

Velocity "diffusion" :

- what are "collisions" for a long-range interaction ?

Periodic jellium model

One-component plasma : electrons in uniform neutralizing background (jellium)
Periodic boundary conditions : cube modulo L

$$\mathbf{k}_{\mathbf{m}} = \frac{2\pi}{L}(m_x, m_y, m_z) \tag{1}$$

$$\tilde{\varphi}(\mathbf{m}) = -\frac{e}{\epsilon_0 k_{\mathbf{m}}^2} \sum_{j \in S} \exp(-\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \mathbf{r}_j), \quad \mathbf{m} \neq \mathbf{0} \tag{2}$$

$$\varphi(\mathbf{r}) = \frac{1}{L^3} \sum_{\mathbf{m}} \tilde{\varphi}(\mathbf{m}) \exp(\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \mathbf{r}) \tag{3}$$

$$\ddot{\mathbf{r}}_l = \frac{e}{m_{\mathrm{e}}} \nabla \varphi_l(\mathbf{r}_l), \quad l \in S = \{1, \ldots N\} \tag{4}$$

with $\varphi_l = \sum_{j \neq l} \ldots$

Classical theory of collisions in plasmas (Berkeley, 1957)

- Gas : successive, brief, near-independent collisions with impact parameter $\lambda_{\mathrm{ma}} \lesssim b \ll n^{-1/3}$ $\Rightarrow$ propagation of initial randomness ("molecular chaos")

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- Plasma : Coulomb is long-range... $\Rightarrow$ relevant scales : $\lambda_{\mathrm{ma}} \ll d \ll \lambda_{\mathrm{D}}$
  - classical distance of minimum approach $\lambda_{\mathrm{ma}} = e^2/(4\pi\epsilon_0 k_{\mathrm{B}} T)$
  - interparticle distance $d = n^{-1/3}$
  - Debye length $\lambda_{\mathrm{D}} = [(\epsilon_0 k_{\mathrm{B}} T)/(ne^2)]^{1/2} = d^{3/2}/(4\pi\lambda_{\mathrm{ma}})^{1/2}$

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- Close encounter $\lambda_{\rm ma} \lesssim b \ll d$ (Rosenbluth, MacDonald & Judd) : Rutherford, two-body (cross-section diverges for $b \to 0$: Coulomb logarithm)

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- Far away $d \lesssim b \sim \lambda_{\mathrm{D}}$ (Gasiorowicz, Neuman & Riddell) : two-body with mean-field tames divergence in plasma limit ($N_{\mathrm{D}} \gg 1$, viz. $d \ll \lambda_{\mathrm{D}}$) (revisited by Balescu and Lenard, 1962)

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- Far away $d \lesssim b \sim \lambda_{\rm D}$ (Gasiorowicz, Neuman & Riddell) : two-body with mean-field tames divergence in plasma limit
- $\Rightarrow$ velocity diffusion coefficient D diverges like $\ln \frac{\lambda_{\rm D}}{\lambda_{\rm ma}} \approx \frac{3}{2} \ln \frac{d}{\lambda_{\rm ma}}$

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- Plasma : Coulomb is long-range...
  Rosenbluth *et al.* : $\lambda_{\rm ma} \lesssim b \ll d$
  Gasiorowicz *et al.*, Balescu : $d \lesssim b$
  accepted... but
  - no model for $b \approx d$ ... though formulæ match
  - simultaneous effect of many interactions instead of separate brief encounters

Linear perturbation to ballistic approximation

... with effective potential

$$
\begin{aligned}
\mathbf{r}_l^{(0)} &= \mathbf{r}_{l0} + \mathbf{v}_l t && (5)\\
\delta\mathbf{r}_l &= \mathbf{r}_l - \mathbf{r}_l^{(0)} && (6)\\
\delta\ddot{\mathbf{r}}_l &= \sum_{j\in S; j\neq l} \mathbf{a}(\mathbf{r}_l - \mathbf{r}_j, \mathbf{v}_j) && (7)\\
\mathbf{a}(\mathbf{r},\mathbf{v}) &= \frac{e}{m_{\mathrm{e}}} \nabla \Phi(\mathbf{r},\mathbf{v}) && (8)\\
\Phi(\mathbf{r},\mathbf{v}) &= -\frac{e}{L^3\epsilon_0} \sum_{\mathbf{m}} \frac{\exp(\mathrm{i}\mathbf{k_m}\cdot\mathbf{r})}{k_{\mathbf{m}}^2 \epsilon(\mathbf{m}, \mathbf{k_m}\cdot\mathbf{v})} && (9)
\end{aligned}
$$

For slow particles, $\epsilon(\mathbf{m}, \mathbf{0}) \approx 1 + (k_{\mathbf{m}}\lambda_{\mathrm{D}})^{-2}$ and $\Phi \approx$ Yukawa potential

Perturbative approximation to trajectories (Picard)

$$\delta\dot{\mathbf{r}}_l(t) = \sum_{j\in S;j\neq l} \delta\dot{\mathbf{r}}_{lj}(0,t) \quad (10)$$

$$\delta\dot{\mathbf{r}}_{lj}(t_1,t_2) = \int_{t_1}^{t_2} \mathbf{a}[\mathbf{r}_l^{(0)}(t') - \mathbf{r}_j^{(0)}(t'), \mathbf{v}_j]\,\mathrm{d}t' \quad (11)$$

Let

$$\mathbf{r}_l^{(0)}(t') - \mathbf{r}_j^{(0)}(t') = \mathbf{b}_{lj} + \Delta\mathbf{v}_{lj}(t'-t_{lj}) \quad (12)$$

with t_{lj} : time of closest approach of the two ballistic orbits,
$\mathbf{b}_{lj} := \mathbf{r}_l - \mathbf{r}_j$ at this time ("impact parameter")

NB for $b \sim \lambda_{\mathrm{D}}$: $\Delta v_{lj} \sim v_{\mathrm{th}}$, deflection duration $\Delta t_{lj} \sim {\omega_{\mathrm{p}}}^{-1}$

Close collisions – strong deflections

Close approach : $b_{lj} \sim \lambda_{\mathrm{ma}}$ ... far from all other particles

$$\ddot{\mathbf{r}}_l = \mathbf{a}(\mathbf{r}_l - \mathbf{r}_j, \mathbf{v}_j) + \sum_{p \in S; p \neq l,j} \mathbf{a}(\mathbf{r}_l - \mathbf{r}_p, \mathbf{v}_p) \tag{13}$$

$$\frac{\mathrm{d}^2(\mathbf{r}_l - \mathbf{r}_j)}{\mathrm{d}t^2} \approx 2\mathbf{a}(\mathbf{r}_l - \mathbf{r}_j) \tag{14}$$

viz. Rutherford collision in centre-of-mass frame

Small deflections by distant particles

Faraway particles :

$$\delta\ddot{\mathbf{r}}_{lj} = \mathbf{a}_{\mathrm{C}}(\mathbf{r}_l - \mathbf{r}_j) \tag{15}$$

$$\mathbf{a}_{\mathrm{C}}(\mathbf{r}) = \frac{\mathrm{i}e^2}{\epsilon_0 m_{\mathrm{e}} L^3} \sum_{\mathbf{m}\neq\mathbf{0}} k_{\mathbf{m}}^{-2}\, \mathbf{k}_{\mathbf{m}} \exp(\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \mathbf{r}). \tag{16}$$

$$\delta\ddot{\mathbf{r}}_{lj}^{(n)} = [\delta\ddot{\mathbf{r}}_{lj}^{(1)} + M_{lj}^{(n-1)} + 2\nabla\mathbf{a}_{\mathrm{C}}(\mathbf{r}_l^{(0)} - \mathbf{r}_j^{(0)}) \cdot \delta\mathbf{r}_{lj}^{(n-1)}] + O(a^3) \tag{17}$$

where $a \sim$ total Coulombian acceleration, and

$$M_{lj}^{(n-1)} = \nabla\mathbf{a}_{\mathrm{C}}(\mathbf{r}_l^{(0)} - \mathbf{r}_j^{(0)}) \cdot [\delta\mathbf{r}_l^{(n-1)} - \delta\mathbf{r}_j^{(n-1)} - 2\delta\mathbf{r}_{lj}^{(n-1)}] \tag{18}$$

$$= \nabla\mathbf{a}_{\mathrm{C}}(\mathbf{r}_l^{(0)} - \mathbf{r}_j^{(0)}) \cdot \sum_{i\in S; i\neq l,j} (\delta\mathbf{r}_{li}^{(n-1)} + \delta\mathbf{r}_{ij}^{(n-1)}) \tag{19}$$

interaction mediated by many particles in Debye sphere

Small deflections by distant particles

Faraway particles : effective potential $\Phi(\mathbf{r}, \mathbf{v}) \simeq \Phi(\mathbf{r}, \mathbf{0})$
Yukawa-like $\Phi_{\mathrm{Y}}(\mathbf{r}) = -e\,(4\pi\,\epsilon_0 \|\mathbf{r}\|)^{-1} \exp(-\|\mathbf{r}\|/\lambda_{\mathrm{D}})$
Deflection is small, to first order

$$\delta\dot{\mathbf{r}}_{lj}(-\infty, +\infty) = \frac{e^2}{4\pi m_{\mathrm{e}}\epsilon_0}\,\mathbf{b}_{lj} \int_{-\infty}^{+\infty} \left[\frac{1}{r^3(t)} + \frac{1}{\lambda_{\mathrm{D}} r^2(t)}\right] \exp[-\frac{r(t)}{\lambda_{\mathrm{D}}}]\,\mathrm{d}t, \tag{20}$$

where $r(t) = \sqrt{b_{lj}^2 + \Delta v_{lj}^2 t^2}$, hence

$$\delta\dot{\mathbf{r}}_{lj}(-\infty, +\infty) = -\frac{2e^2}{4\pi m_{\mathrm{e}}\epsilon_0 \Delta v_{lj}}\,\frac{h(b_{lj})}{b_{lj}^2}\,\mathbf{b}_{lj}, \tag{21}$$

where

$$h(b) = \int_0^{\pi/2} \left[\cos\theta + \frac{b}{\lambda_{\mathrm{D}}}\right] \exp[-\frac{b}{\lambda_{\mathrm{D}}\cos\theta}]\,\mathrm{d}\theta. \tag{22}$$

Velocity diffusion model

In hot plasma, many slow deflections occur simultaneously
but they are small

$$\langle \|\delta \dot{\mathbf{r}}_l(t)\|^2 \rangle = \sum_{j \in S; j \neq l} \langle \|\delta \dot{\mathbf{r}}_{lj}(0,t)\|^2 \rangle \quad (23)$$

$$= \int_{b_{lj}} \int_{\mathbf{v}_j} \frac{e^4 2\pi \, \Delta v_{lj} \, t \, b_{lj}}{(2\pi m_{\mathrm{e}} \epsilon_0 \Delta v_{lj})^2} \left(\frac{h(b_{lj})}{b_{lj}} \right)^2 f(\mathbf{v}_j) \, \mathrm{d}^3\mathbf{v}_j \, \mathrm{d}b_{lj}$$

$$\sim C \int_{\lambda_{\mathrm{ma}}}^{\infty} \frac{(h(b))^2}{b} \, \mathrm{d}b \quad (24)$$

Gasiorowicz *et al.* : $h(b) = 1(0 < b \leq \lambda_{\mathrm{D}})$ ... Coulomb log & cut-off

New : smooth h, with $1 - cb \leq h(b) \leq 1$ for $0 \leq b \leq \lambda_{\mathrm{D}}$, fast decay as $b/\lambda_{\mathrm{D}} \to \infty$, matches with Rosenbluth *et al.*

Summary

Benefits :

- single description for $\lambda_{\mathrm{ma}} \lesssim b < \infty$
- simultaneous effects of slow small deflections instead of only brief brutal ones
- cooperation : two-body effect mediated by many particles

Open issue :

- Debye shielding good for slow particles... but not for fast ones

From Coulomb to Debye

$$\Phi(\mathbf{r}, \mathbf{v}) = -\frac{e}{L^3 \epsilon_0} \sum_{\mathbf{m}} \frac{\exp(\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \mathbf{r})}{k_{\mathbf{m}}^2 \epsilon(\mathbf{m}, \mathbf{k}_{\mathbf{m}} \cdot \mathbf{v})}$$

Periodic jellium model

One-component plasma : electrons in uniform neutralizing background (jellium)
Periodic boundary conditions : cube modulo L

$$\begin{aligned}
(2) \qquad \tilde{\varphi}(\mathbf{m}) &= -\frac{e}{\epsilon_0 k_{\mathbf{m}}^2} \sum_{j \in S} \exp(-\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \mathbf{r}_j), \quad \mathbf{m} \neq \mathbf{0} \\
(3) \qquad \varphi(\mathbf{r}) &= \frac{1}{L^3} \sum_{\mathbf{m}} \tilde{\varphi}(\mathbf{m}) \exp(\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \mathbf{r}) \\
(4) \qquad \ddot{\mathbf{r}}_l &= \frac{e}{m_{\mathrm{e}}} \nabla \varphi_l(\mathbf{r}_l), \quad l \in S = \{1, \ldots N\}
\end{aligned}$$

with $\varphi_l = \sum_{j \neq l} \ldots$

Smoothing the potential

Let $\lambda_{\mathrm{ma}} \ll b_{\mathrm{smooth}} \ll \lambda_{\mathrm{D}}$ and $\mathcal{M}_{\mathrm{smooth}} := \{\mathbf{k} : \|\mathbf{k}\| b_{\mathrm{smooth}} \leq 1\}$.
Let $\Delta \mathbf{r}_j(t) := \mathbf{r}_j(t) - \mathbf{r}_{j0} - \mathbf{v}_j t$ and linearize for $k \|\Delta \mathbf{r}_j\| \ll 1$:

$$\tilde{\varphi}_{\mathrm{lin}}(\mathbf{m}, t) = -\sum_{j=1}^{N} \frac{e}{\epsilon_0 k_{\mathbf{m}}^2} \, \mathrm{e}^{-\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot (\mathbf{r}_{j0} + \mathbf{v}_j t)} \, [1 - \mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \Delta \mathbf{r}_j(t)] \qquad (25)$$

Laplace transform : $\widehat{f}(\omega) = \int_0^\infty f(t) \, \mathrm{e}^{\mathrm{i}\omega t} \, \mathrm{d}t$ (with ω complex)

$$\varphi_{\mathrm{lin}}(\mathbf{m}, \omega) = -\sum_{j=1}^{N} \frac{e}{\epsilon_0 k_{\mathbf{m}}^2} \, \mathrm{e}^{-\mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \mathbf{r}_{j0}} \, [\frac{\mathrm{i}}{\omega - \mathbf{k}_{\mathbf{m}} \cdot \mathbf{v}_j} - \mathrm{i}\mathbf{k}_{\mathbf{m}} \cdot \Delta \mathbf{r}_j(\omega - \mathbf{k}_{\mathbf{m}} \cdot \mathbf{v}_j)],$$

$$\begin{aligned} \Delta \ddot{\mathbf{r}}_j &= \frac{e}{m_{\mathrm{e}}} \nabla \varphi_{\mathrm{lin}}(\mathbf{r}_j) \\ &= \frac{\mathrm{i}e}{L^3 m_{\mathrm{e}}} \sum_{\mathbf{n},\, k_{\mathbf{n}} b_{\mathrm{smooth}} \leq 1} \mathbf{k}_{\mathbf{n}} \, \tilde{\varphi}_{\mathrm{lin}}(\mathbf{n}, t) \exp[\mathrm{i}\mathbf{k}_{\mathbf{n}} \cdot (\mathbf{r}_{j0} + \mathbf{v}_j t)] \qquad (26) \end{aligned}$$

Smoothing the potential

Compatibility condition :

$$
\begin{aligned}
& k_{\mathbf{m}}^2 \varphi_{\mathrm{lin}}(\mathbf{m}, \omega) \\
& -\frac{\omega_{\mathrm{p}}^2}{N} \sum_{\mathbf{n},\, k_{\mathbf{n}} \in \mathcal{M}_{\mathrm{smooth}}} \mathbf{k_m} \cdot \mathbf{k_n} \sum_{j=1}^{N} \frac{\varphi_{\mathrm{lin}}(\mathbf{n}, \omega + \mathbf{k_{n-m}} \cdot \mathbf{v}_j)}{(\omega - \mathbf{k_m} \cdot \mathbf{v}_j)^2} \mathrm{e}^{\mathrm{i}(\mathbf{k_n} - \mathbf{k_m}) \cdot \mathbf{r}_{j0}} \\
= \;& k_{\mathbf{m}}^2 \varphi_{\mathrm{lin}}^{(\mathrm{bal})}(\mathbf{m}, \omega) \qquad (27)
\end{aligned}
$$

Linear, with

$$
\varphi_{\mathrm{lin}}^{(\mathrm{bal})}(\mathbf{m}, \omega) = -\sum_{j \in S} \frac{\mathrm{i}e}{\epsilon_0 k_{\mathbf{m}}^2} \frac{\exp[-\mathrm{i}\mathbf{k_m} \cdot \mathbf{r}_{j0}]}{\omega - \mathbf{k_m} \cdot \mathbf{v}_j} \qquad (28)
$$

the particles ballistic contribution to the total potential.

Smoothing the ballistic background

$$\sum_{l \in S} \bullet = \iint \bullet\, f(\mathbf{r}, \mathbf{v})\, \mathrm{d}^3\mathbf{r}\ \mathrm{d}^3\mathbf{v} + W(\bullet) \tag{29}$$

Coarse-graining $N \to \infty$: neglect W. Then

$$\epsilon(\mathbf{m}, \omega)\, \varphi_{\mathrm{cg}}(\mathbf{m}, \omega) = \varphi_{\mathrm{lin}}^{(\mathrm{bal})}(\mathbf{m}, \omega), \tag{30}$$

$$\epsilon(\mathbf{m}, \omega) = 1 - \frac{e^2}{L^3 m_{\mathrm{e}} \epsilon_0} \int \frac{f_0(\mathbf{v})}{(\omega - \mathbf{k_m} \cdot \mathbf{v})^2}\ \mathrm{d}^3\mathbf{v} \tag{31}$$

Smoothing the ballistic background

Inverse Fourier-Laplace... $\Phi = \sum_{j \in S} \delta\Phi_j$ with

$$\delta\Phi_j(\mathbf{r}) = \delta\Phi(\mathbf{r} - \mathbf{r}_j(0) - \dot{\mathbf{r}}_j(0)t, \dot{\mathbf{r}}_j(0)) \tag{32}$$

after transient, with shielded Coulomb potential

$$\delta\Phi(\mathbf{r}, \mathbf{v}) = -\frac{e}{L^3 \epsilon_0} \sum_{\mathbf{m} \neq \mathbf{0}} \frac{\exp(\mathrm{i}\mathbf{k_m} \cdot \mathbf{r})}{k_{\mathbf{m}}^2 \, \epsilon(\mathbf{m}, \mathbf{k_m} \cdot \mathbf{v} + \mathrm{i}\varepsilon)} \tag{33}$$

where

$$\epsilon(\mathbf{m}, \mathbf{k_m} \cdot \mathbf{v} + \mathrm{i}\varepsilon) \approx 1 + \frac{k_{\mathrm{D}}^2}{k_{\mathbf{m}}^2} \tag{34}$$

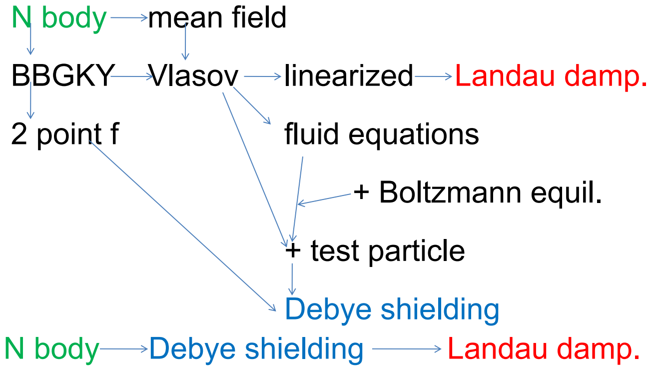
Smoothing the sources

Smoothing sources in ballistic potential $\varphi_{\rm lin}^{\rm (bal)}$ too

$$\varphi_{\rm cg}^{\rm (bal)}(\mathbf{m}, \omega) = -\frac{\mathrm{i}e}{\epsilon_0 k_{\mathbf{m}}^2} \int \frac{\tilde{f}(\mathbf{m}, \mathbf{v})}{\omega - \mathbf{k}_{\mathbf{m}} \cdot \mathbf{v}} \, \mathrm{d}^3\mathbf{v} \tag{35}$$

viz. Landau integral for field evolution in Landau damping or growth.

1. Direct path from N-body dynamics to Debye shielding and Landau damping



Fast: 3 REVTeX pages only

Intuitive interpretation of Debye shielding

N-body dynamics intuition for shielding

Newton's second law & Coulomb potential
Picard iteration technique for $\dot{\mathbf{X}} = f(\mathbf{X})$:

- $\mathbf{X}^{(0)}$ chosen approximation
- $\dot{\mathbf{X}}^{(n+1)} = f(\mathbf{X}^{(n)})$

No probabilistic setting
The acceleration of a particle due to another one has
a part mediated by all other particles

Deflections $\rightarrow$ lower apparent charge of deflector
Each particle can be shielded by all other ones,
while all particles are in uninterrupted motion

Debye shielding and collisional transport are two sides of the same coin :
deflections

Further reading

- D.F. Escande, Y. Elskens, F. Doveil, *J. Plas. Phys.* 81 (2015) 305810101 (9 pages)
- D.F. Escande, F. Doveil, Y. Elskens, *Plas. Phys. Contr. Fus.* (at press) hal-01165794